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Multivariate Approximation: Theory and Applications
An Overview

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Abstract

The fourth conference on "Multivariate Approximation: Theory and Applications" was held April 26 to May 01, 2007 at Cancun, Mexico. This introductory paper describes the main topics in Approximation Theory presented during this conference, ranging from theoretical aspects to numerous applications.

1 Introduction

The fourth conference on "Multivariate Approximation: Theory and Applications" was held April 26 to May 01, 2007 at Cancun, Mexico (Fig. 1). The three first conferences of this series were: the First International Conference on Scattered Data Fitting, organized by A. Le Méhauté, L.L. Schumaker and L. Traversoni in March 1995, at Cancun, Mexico (the proceedings appeared in the Journal of Computational and Applied Mathematics 73, 1996), the Second International Conference on Multivariate Scattered Data Fitting, organized by P. Gonzalez-Casanova, A. Le Méhauté, L.L. Schumaker and L. Traversoni in April 1999 at Puerto Vallarta, Mexico, and the Third Multivariate Approximation: Theory and Applications conference organized by C. Gout, C. Rabut and L. Traversoni in April 2003 at Cancun, Mexico (the proceedings appeared in Numerical Algorithms 39, 2005).

The conference was attended by 65 participants from 18 countries, and 47 communications were presented. The web site of the conference is
http://www.univ–pau.fr/∼cgout/mata2007.

This issue of Numerical Algorithms includes 12 papers based on these talks. All these papers have been refereed according to the usual rules of Numerical Algorithms, and we want to express here our warm thanks to all the referees of this issue.

2 Main themes of the conference

The conference dealt with pure and applied approximation theory and related areas. The primary topics of interest were polynomials, curves, surfaces, and volumes, such as splines, meshes, subdivision surfaces as well as algorithms to generate, analyze, and manipulate them.

- In the area of computation or storage, subdivision methods, data reduction methods and wavelets are quite recent tools. Nevertheless, corresponding methods are already well spread out. They provided and will in the future give lots of new possibilities, for

theoretical results as well as for various applications, such as jpeg image compression (which uses tensor product wavelets), or large scale data handling in meteorology. New theoretical tools are under development, e.g., those using quaternions, or even quaternion wavelets, with applications in trajectory prediction, robotics, satellites, etc. For example, the quaternion wavelet transform has achieved much attention in recent years as a new multiscale analysis tool for geometric image structures.

- In the analytical framework, multivariate approximation and associated data reduction are of great interest, and terrain modelling, meteorology, hydrology for example are calling for new theoretical results in order to improve and extend the methods presently used. In geometry, 3D CAGD methods are mainly based on multivariate approximation results, and many people would like improvements in terms of definition, flexibility (e.g., scattered data instead of gridded data, speed of computation), or of new tools such as developable surfaces or wavelets based on non-separable functions.

- Elsewhere, in recent years, image processing constitutes a theme where many fields in numerical analysis focus: from deformable models to level-set methods and fast marching methods, with many applications to image reconstruction, segmentation, or noise removal.

The conference talks, as well as the papers in this issue, presented theoretical results as well as applications in all these three areas, showing the dynamics of research in multivariate approximation.

3 Short overview of the conference

In this section, we describe the main topics of the conference along with (some of) their corresponding communications.

- **Approximation by splines:** Splines are still a very active subject for research in approximation, both from a theoretical point of view and for applications, as we can see by the number of talks and papers devoted to splines:

  In curve and surface modelling, Hoffmann [24] provides a general framework for several spline curve types with shape parameters. In [3], the authors work on parametric curves (degree 3 or 5 splines) minimizing the L_p-nom of the second derivative, with a possibility to obtain convexity properties. An interesting approach has been presented in [37]: in many problems of geophysical interest, one has to deal with data that exhibit complex fault structures. In these circumstances, due to the presence of large and rapid variations in the data, attempting a fitting using conventional approximation methods necessarily leads to instability phenomena or undesirable oscillations which can locally and even globally hinder the approximation. As shown in this talk, a correct approach to get a good approximant consists, in effect, in applying first a segmentation process to precisely define the locations of large variations and faults and exploiting then a standard approximation technique. Hernandez [23] proposes an advancing front method for generating an isotropic triangular mesh on a regular parametric surface. In [13], [14] and [15], the authors discuss an extension of the algorithm from the case of parametric surfaces with a priori given closed-form parametrization to the case when only interpolation data is provided on a rectangular-grid or scattered-point set in the parametric domain. Polyharmonic splines are also a tool that permits many applications as shown in [5] and [36] where associated multiresolution analysis and properties of the operators obtained, convergence results and applications are given. Other results are given in Ablamowicz [1] where computational algebraic geometry methods for curves and surfaces are presented. In [25], the authors present the construction and the efficient evaluation of the Lagrange interpolating polynomial over a lattice on a simplex using barycentric coordinates.

Neamtu [32] recalls some theory of bivariate splines which fails completely when one wants to define smooth surfaces of arbitrary topological genus. So, he addresses the reasons of this serious gap in spline theory and discusses a possible remedy.

In 1988, Worsey and Piper constructed a trivariate macro-element based on C^1 quadratic splines defined over the Powell-Sabin split of a tetrahedron. Unfortunately, their element can only be used with tetrahedral partitions that satisfy some very restrictive geometric constraints. Schumaker [38] shows that, by further refining their split, it is possible to construct a macro-element (based on the same spline space) that can be used with arbitrary tetrahedral partitions.

Application of cubic splines : in [7], the authors presented how they used complex cubic splines to control the position, orientation and velocity of a differential mobile robot.

- **Image processing:** As presented by Mørken [31], the ability to capture images of the interior of the human body has revolutionized both medical diagnosis and therapy. Today, a number of different imaging technologies are available, such as Computed Tomography (CT) (see [21]), Magnetic Resonance Imaging(MRI), Positron Emission Tomography (PET) ultrasound, X-ray, photo and video. The first three techniques provide 3D samples on uniform grids; ultrasound can give both 2D and 3D images, while X-ray gives 2D images, but is also the basis for CT imaging. Although these technologies have already radically changed many aspects of medicine, with for instance the emergence of minimally invasive surgery, new surgery techniques based on imaging are likely to further transform both diagnosis and therapy in even more profound ways. A fundamental challenge in medical imaging is to combine data from different imaging techniques to produce a synthetic image that provides more information than any of the individual images. This is the image fusion problem, which in turn is dependent on the image registration problem, i.e., the problem of aligning two images (2D or 3D) in a common coordinate system. In general, these are extremely challenging problems, both mathematically and computationally. Moreover, Le Guyader and Vese [28] & [45], Forcadel et al. [18], Gout and Le Guyader [20] have presented various different methods. In particular, in [18], the authors propose a segmentation method based on the Generalized Fast Marching Method (GFMM) based on the classical Fast Marching Method (FMM) (a very efficient method for front evolution problems with normal velocity of constant sign). The GFMM is an extension of the FMM and removes the sign constraint by authorizing time-dependent velocity with no restriction on the sign. In their modelling, the velocity is borrowed from the well-known Chan-Vese model for segmentation. The GFMM offers a powerful and computationally efficient algorithm, with many other applications. As stressed by Steidl [41], methods for image restoration that respect edges and other important features are of fundamental importance in digital image processing; she proposes a new method for the restoration of images containing rotated (linearly transformed) rectangular shapes which avoids the round-off effect at vertices produced by known edge-preserving denoising techniques. To do so, she employs a diffusion-inspired coupling of the wavelet channel which guarantees an approximation with an excellent degree of rotation invariance.

- **Multiscale and wavelets modelling:** Dadourian [11] presented a nonlinear subdivision scheme, worked out as a sum of the linear part and a nonlinear part. Under natural conditions, this scheme (and so the associated multi-resolution) is convergent and stable. Angela Kunoth [27] addresses fast multiscale PDE-solvers for control problems constrained by elliptic PDEs with distributed as well as Dirichlet boundary control. She stresses extra difficulties for PDE-constrained control problems with additional inequality constraints on the control. Rossini [36] showed how to build various polyharmonic B-splines and associated multi-resolution analysis. Mhaskar [30] constructs a multiscale tight frame based on an arbitrary orthonormal basis for the L^2 space of an arbitrary sigma-finite measure space.

- **Radial basis functions:** Not surprisingly, many people use or study this tool, now extremely popular with engineering and other applications: Martin Buhmann [6] studies the impact of various parameters (such as the usual parameter c), in terms of the conditioning problems, the limit of the interpolating function, or the form of the functions. Gonzalez-Casanova [19] deals with the position of the centres: he derives, in an adaptive way, subdomains in accordance with the knot insertion-removal technique, and so reduces the computational cost and solves problems with a large number of knots. D^m (or polyharmonic) splines are particular, but most widely used, RBF functions and allow important theoretical developments. They have been specifically studied by Lopez de Silanes [2] (extension of a bound for functions in Sobolev spaces, with special attention on the impact of the smoothing parameter), Bozzini [5] (large number of data, highly unevenly distributed), Rossini [36] (building multiresolution analysis), Saint Guirons [37] (a segmentation process for approximation of surfaces with faults and/or rapidly varying data).

- **Theoretical mathematical subjects** (connected to Approximation Theory): in that rubric, let us mention here a study on Cartan-Dieudonné's theorem [10], another one on the Runge phenomenon [34], and the one on a new approach for numerical integration on compact homogeneous spaces, using energy criteria [12]. Waldron [46] presented various frames designed to respect symmetries of the underlying spaces; in particular he presented the class of harmonic frames of n vectors in C^d that are generated by an abelian subgroup of symmetries.

- **Quaternions:** Here again we can see both the theoretical approach and the application-oriented one: new results in discrete Clifford analysis were presented by Van de Voorde [44], a new Hilbert transform was introduced by de Knock [16] in a metric dependent Clifford setting. Xu [47] reviews work done on the Quaternionic Wavelet Transform and gives some applications of it, such as color image processing, object recognition, optical flow estimation and stereo matching. Applications of quaternions are presented by Reséndiz [35] (for building solids), by Falcon [17] (3D motion estimation using robot omnidirectional vision), by Sprössig [40] (fluid dynamics and elasticity). Traversoni [43] presented different approaches of using quaternionic wavelets for movement detection and recognition.

- **Applications:** As expected, many applications were presented during the conference. Let us mention some of them: in the medical domain are the magneto-encephalography (Pitolli [33]), the tomography, echography, the breathing sounds (Charleston [8]). Image processing (decomposition, reconstruction of images, image compression), contour detection and determination of some characteristics of images, (Steidl [41]), hand-written character recognition (Cisneros [9], Toscano [42]), geodesic applications. Applications of quaternions: robotics [4], 3D trajectories estimation [17], fluid dynamics and elasticity [40].

  Simulation is also an important application of modelisation and approximation. Hector [26] presented a simulation of a viscous pump.

  Obviously there are also many commercial applications that were not presented during the conference. The variety and the number of applications (actually we could quite say all concrete applications in 2D or 3D) show the importance of the present theoretical research. We believe that applications based on multivariate approximation is a research area which will continue to grow in the short and long time future.

4 Conclusion

The wide spectrum of theoretical talks as well as the ones on applications show that there is still a lot of work to be done in that domain. Whereas it seems that univariate approximation is now well understood, there are still many issues that are specific to multivariate cases and that need new results. Actually image processing, detection of faults or contours, pattern (or character) recognition are specific to the multivariate case and still need lot of work to be more efficient. In particular, medical imaging is a challenge to get precise information from noisy and specific measurements (such as in PET, where the measures are directional integrals of the quantity to be reconstructed). Here, there is in addition the challenge of developing fast methods for handling a large number of (three-dimensional) data, in order to be able to get live information for surgery, or even simply to be able to focus the imaging process on some parts during the data caption. Wavelets can no longer be treated only as tensor products of univariate wavelets, Clifford algebra and quaternions are considered by some of the participants as a promising tool, especially for all spacial-temporal movements. We expect that future work will greatly improve the quality and the precision of the results obtained.

5 Talks of the conference

As references, we give here the title of every talk of the conference. Those in boldface are published in the form of a paper in this issue.

References

[1] Rafal Ablamowicz, Computational algebraic geometry methods for curves and surfaces.

[2] Rémi Arcangéli, Maria Cruz Lopez de Silanes, Juan José Torrens, An extension of a bound for functions in Sobolev spaces and its application to (m, s) spline interpolation and smoothing.

[3] Philippe Auquiert, Olivier Gibaru, Eric Nyiri, C^1 and C^2-continuous polynomial parametric L_p splines ($p \geq 1$).

[4] Eduardo Bayro Corrochano, Applications of quaternion wavelet transform: for robotic vision.

[5] Mira Bozzini, A local method for large and unevenly distributed data.

[6] Martin Buhmann, Radial basis function interpolation with parameters.

[7] Heriberto Casarrubias, Humberto Sossa, Ricardo Barrón, Using cubic spline for the velocity planning in mobile robots.

[8] Sonia Charleston-Villalobos, J. Cruz-García, R. González-Camarena, T. Aljama-Corrales, Forward and inverse problems for determining computer simulated respiratory sound sources.

[9] Hermenegildo Cisneros, Ricardo Barrón, Benjamín Cruz, Isolated character recognition by using Support Vector Machines with a kernel of radial base.

[10] Benjamín Cruz, Ricardo Barrón, Jesús López & Humberto Sossa, Some formal aspects and applications of Cartan Dieudonné theorem.

[11] **Karine Dadourian, Jacques Liandrat, Analysis of some bivariate non-linear interpolatory subdivision schemes and associated multiresolution.**

[12] **Steve B. Damelin, A walk through energy, discrepancy, numerical integration and group invariant measures on measurable subsets of Euclidean space.**

[13] Lubomir T. Dechevsky, Expo-rational B-splines of a complex variable and their applications.

[14] Lubomir T. Dechevsky, Børre Bang, Xian Feng Bu, Arne Lakså, Variational calculus and multigrid dynamical programming on expo-rational surfaces and volume deformations: theory and applications.

[15] Lubomir T. Dechevsky, Arne Lakså, Børre Bang, Generalized expo-rational B-splines.

[16] Bram De Knock, F. Brackx, H. De Schepper, A new Hilbert transform in a metric dependent Clifford setting.

[17] Luis Eduardo Falcon and Eduardo Bayro-Corrochano, Radon transform and harmonical analysis for 3D motion estimation using robot omnidirectional vision.

[18] **Nicolas Forcadel, Carole Le Guyader and Christian Gout, Image segmentation using a generalized fast marching method.**

[19] Pedro González-Casanova, José Antonio Muñoz Gómez and Gustavo Rodríguez Gómez, A dynamical node adaptive RBF method to solve PDE problems.

[20] **Christian Gout and Carole Le Guyader, Geodesic active contour under geometrical conditions: Theory and 3D applications.**

[21] Alexander Grebennikov, J. G. Vázquez Luna, T. Valencia Perez, M. Najera Enriquez, Rotating projection algorithm of image reconstruction and application for computer tomography.

[22] Klaus Gürlebeck, On monogenic approximations of solutions of the Lamé system.

[23] **Victoria Hernandez, Pedro L. del Agel and Jorge Estrada, Umbrella based triangulation of regular parametric surfaces.**

[24] Miklos Hoffmann and Imre Juhasz, General formulae of B-spline curves with shape parameters.

[25] **Gasper Jaklic, Jernej Kozak, Marjeta Krajnc, Vito Vitrih, Emil Zagar, Lattices on simplices.**

[26] L. Héctor Juarez, Numerical Simulation of a Viscous Pump by a High Order Finite Element Method.

[27] **Angela Kunoth, Fast multiscale methods for PDE-constrained control problems.**

[28] Carole Le Guyader and Luminita Vese, Self-repelling snakes for topology-preserving segmentation model.

[29] Jeremy Levesley, The order of approximation of transport equations by the kinetic equation.

[30] H.N. Mhaskar, Diffusion polynomial frames on metric measure spaces.

[31] Knut Mørken, Mathematical and computational challenges in some applications of medical imaging.

[32] Mike Neamtu, Splines for surfaces of arbitrary topology.

[33] Francesca Pitolli, Sparse approximation for source separation in the magnetoencephalography inverse problem.

[34] Christophe Rabut, How to reduce Runge phenomenon.

[35] Rafael Reséndiz, Building solids using quaternions.

[36] **Milvia Rossini, Polyharmonic B-splines.**

[37] **Anne-Gaëlle Saint-Guirons, Christian. Gout, Carole Le Guyader and Lucia Romani, Surface approximation of surfaces with fault(s) and/or rapidly varying data, using a segmentation process, $\mathbf{D}^m$-splines and the finite element method.**

[38] **Larry Schumaker, A $\mathbf{C}^1$ quadratic trivariate macro-element space defined over arbitrary tetrahedral partitions.**

[39] Frank Sommen, The Dirac equation for bivector mass.

[40] Wolfgang Sproessig, Hyperholomorphic functions in elasticity and fluid dynamics.

[41] **Gabriele Steidl, Diffusion filters and wavelets.**

[42] Karina Toscano, Ricardo Barrón, Humberto Sossa, Hand-written character recognition using spline wavelets.

[43] Leonardo Traversoni, Wavelets and movements: different approaches.

[44] Liesbet Van de Voorde, H. De Schepper, Frank Sommen, Some results in discrete Clifford analysis.

[45] Luminita Vese, Variational image decomposition models into cartoon and texture.

[46] **Shayne Waldron, Tight frames and their symmetries.**

[47] Yi Xu, QWT: Retrospective and New Applications.

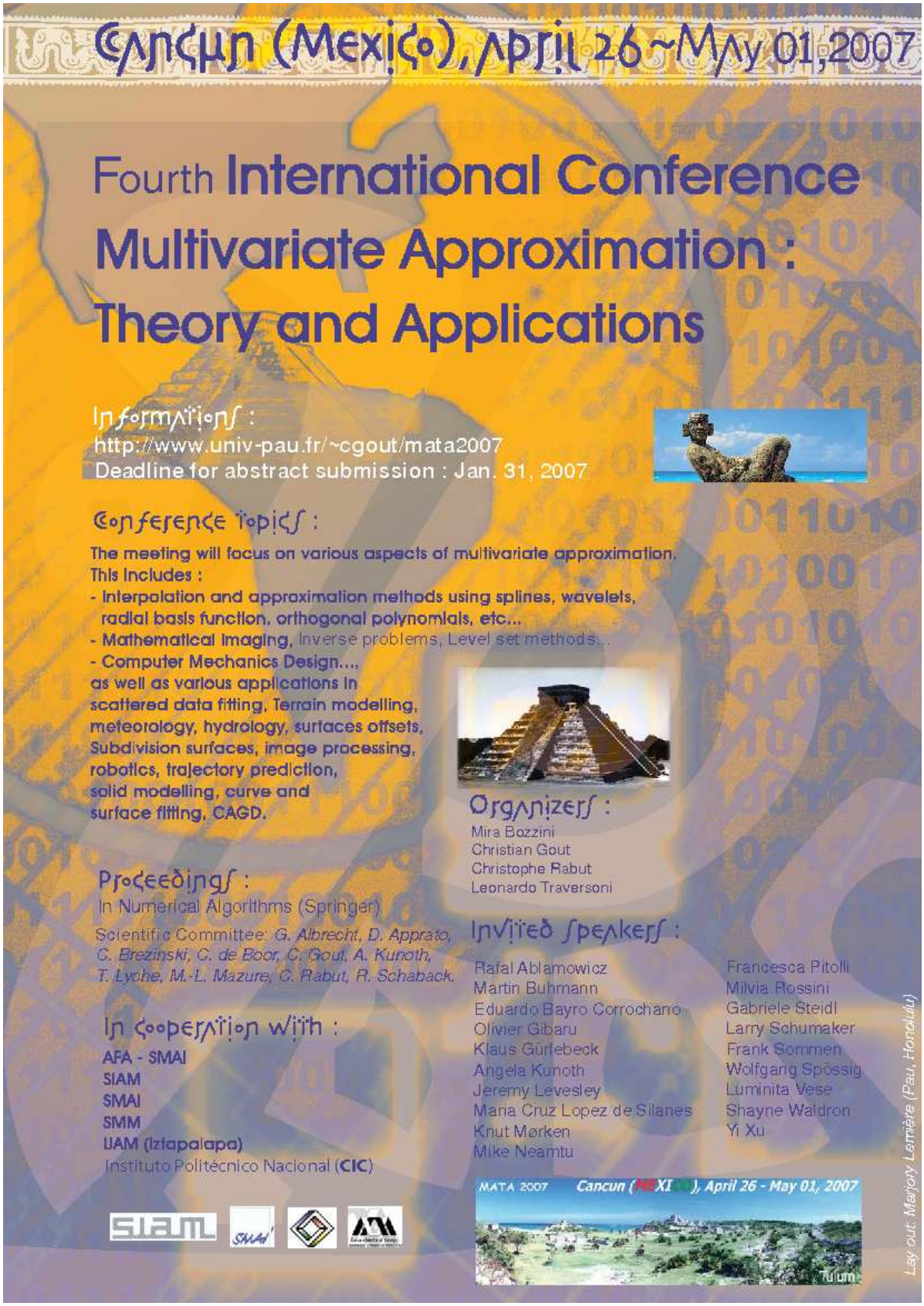

Figure 1: Poster of the conference.